High-Performance Algorithm for the Coupled Conductive-Radiative Heat Transfer Problems

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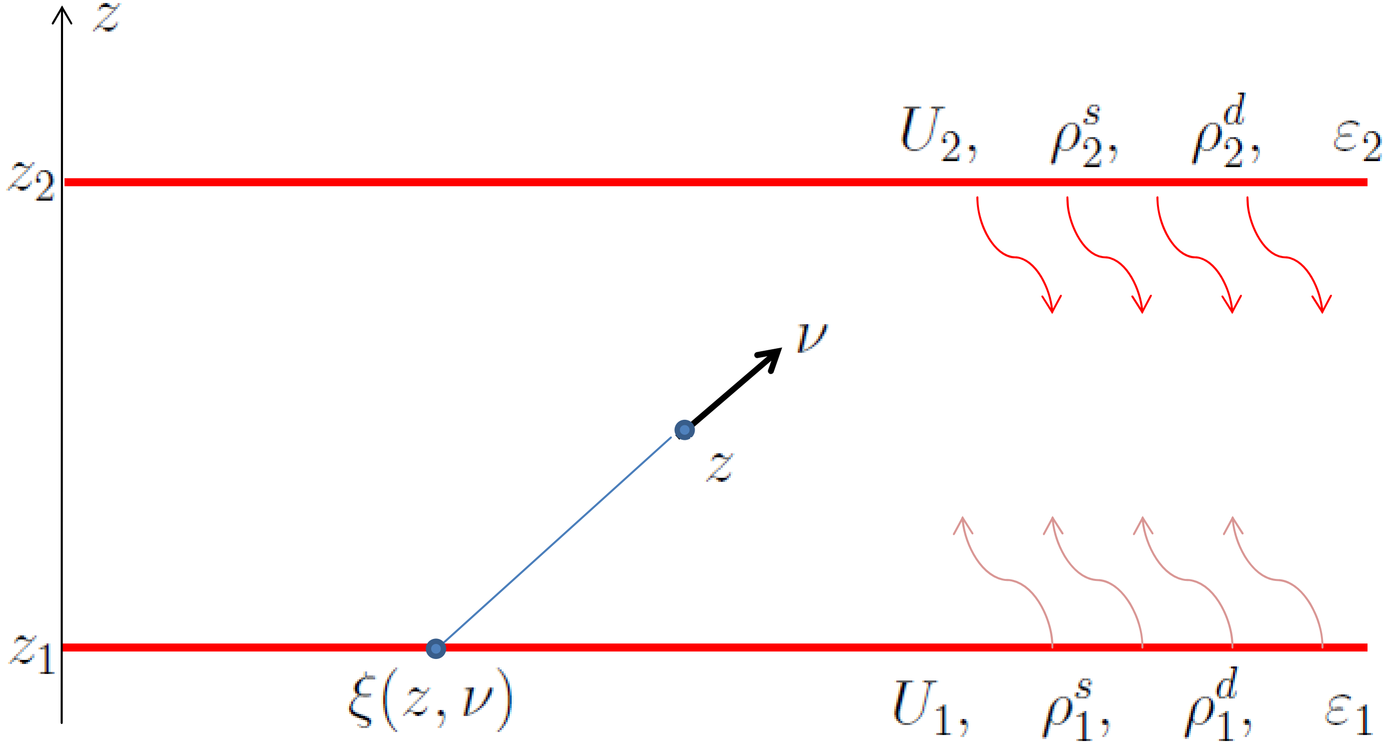


$$X = (z_1, z_2) \times \{[-1, 0) \cup (0, 1]\}.$$

$$\Gamma^- = \{\{z_1 \times (0, 1]\} \cup \{z_2 \times [-1, 0)\}\}$$

Radiative heat transfer equation:

$$\nu f_z(z,\nu) + f(z,\nu) = c\int_{-1}^{1} p(\nu,\nu')f(z,\nu')d\nu' + (1-c)u^4(z), \qquad (1)$$

$$f(z_i,\nu) = h(z_i) + (Bf)(z_i,\nu), \quad i = 1,2, \quad (z_i,\nu) \qquad (2)$$

$$h(z_i) = \varepsilon_i U_i^4, \quad (Bf)(z_i,\nu) = \rho_i^s f(0,-\nu) + 2\rho_i^d \int_0^1 f(0,-\mathrm{sgn}(\nu)\nu)\nu d\nu.$$

Conductive heat transfer equation:

$$u''(z) = \frac{1}{2N_c}\left(\int_{-1}^{1} f(z,\nu)\nu d\nu\right)' \qquad (3)$$

$$u(0) = U_1, \quad u(d) = U_2. \qquad (4)$$

Iterative procedure

$$\tilde{f}_0(z,\nu) = 0 \longrightarrow u_0(z) = \frac{U_2 - U_1}{d} z + U_1 \xrightarrow{(1),(2)} \tilde{f}_1(z,\nu) \xrightarrow{(3),(4)}$$

$$\longrightarrow \tilde{u}_1(z) \longrightarrow u_1(z) = \alpha \tilde{u}_1(z) + (1-\alpha) u_0(z) \longrightarrow \ldots.$$

$$\longrightarrow \tilde{u}_k(z) \longrightarrow u_k(z) = \alpha \tilde{u}_k(z) + (1-\alpha) u_{k-1}(z)$$

Consider the following expression

$$(Tf)(z,\nu) = (Bf)(\xi(\nu),\nu)\exp\left(-\frac{z-\xi(\nu)}{\nu}\right) + (ASf)(z,\nu),$$

here

$$(A\varphi)(z,\nu) = \frac{1}{\nu}\int\limits_{\xi(\nu)}^{z} \exp\left(-\frac{z-z'}{\nu}\right)\varphi(z',\nu)dz',$$

$$(S\varphi)(z,\nu) = c\int_{-1}^{1} p(\nu,\nu')\varphi(z,\nu')d\nu',$$

Solvability of the problem (1),(2)

Theorem 1. *Assuming that the inequalities* $\|B\| \leq 1$ *and* $c < 1$ *hold, there exists a unique solution of the problem (1),(2) that can be found in the form of the Neumann series*

$$f(z,\nu) = \sum_{k=0}^{\infty} (T^k f_0)(z,\nu) \tag{5}$$

converging in the norm of $C_b(X)$.

Recursive relations based on the Monte Carlo method:

$$u(z) = \frac{1}{2N_c} \int_0^z \int_{-1}^1 f(\zeta, \nu) d\zeta d\nu + C_1 z + C_2 \tag{10}$$

$$a(z) = \frac{1}{2N_c} \int_0^z \int_{-1}^1 f_N(\zeta, \nu) d\zeta d\nu \approx \frac{z}{M N_c} \sum_{k=1}^{M} \overline{f}_N(z_k, \nu_k),$$

Recursive algorithm based on the Monte Carlo method

$$f_N(z,\nu) = \sum_{n=0}^{N} (T^n f_0)(z,\nu). \tag{7}$$

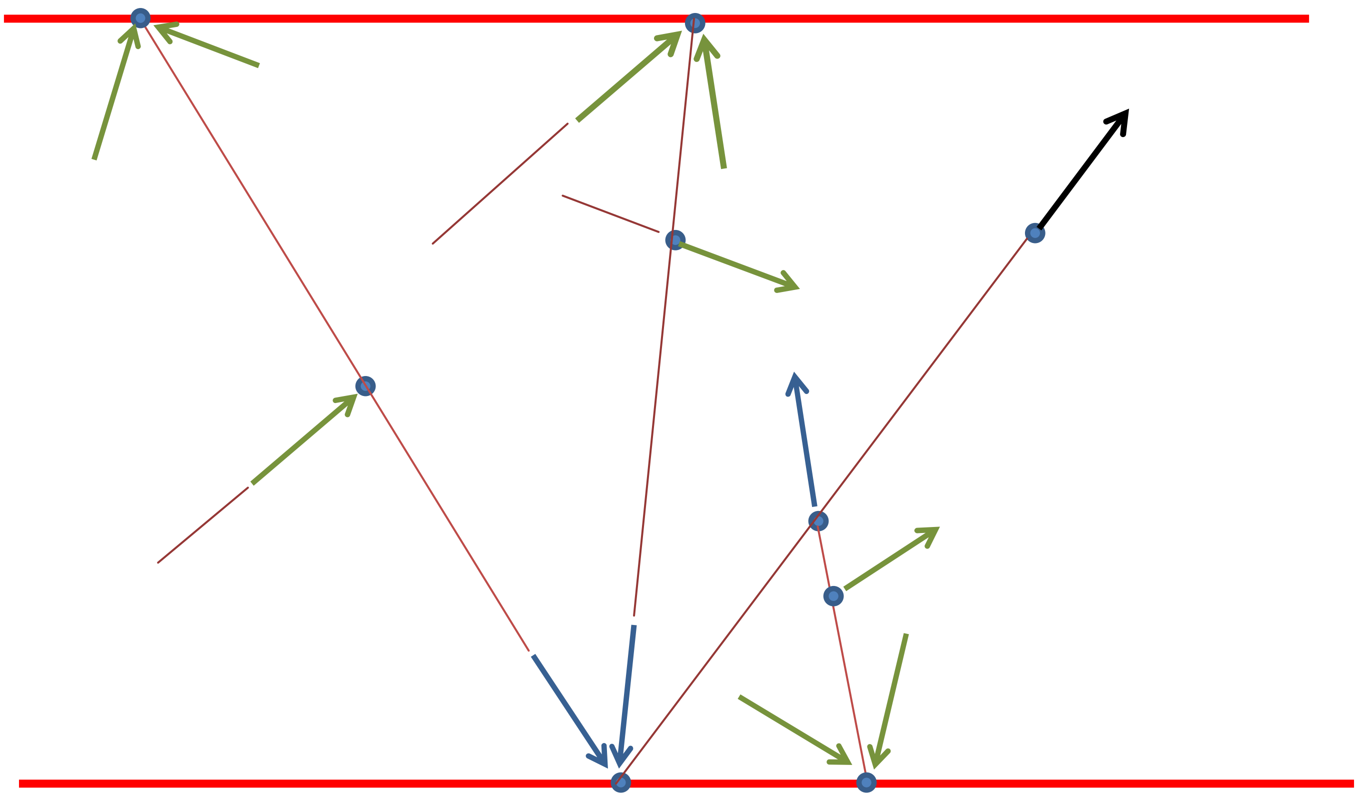
$$f_n(z,\nu) = (T f_{n-1})(z,\nu) + f_0(z,\nu), \quad n = 1, 2, ..., N. \tag{8}$$

$$f_n(z,\nu) \approx \overline{f}_n(z,\nu) = \frac{1}{M}\sum_{k=1}^{M} s_k(z,\nu), \quad \overline{f}_0(z,\nu) = f_0(z,\nu), \tag{9}$$

$$s_k(z,\nu) = \left(B\overline{f}_{n-1}\right)(\xi(\nu),\nu)\exp\left(-\frac{z-\xi(\nu)}{\nu}\right) +$$

$$+c\left(1-\exp\left(-\frac{z-\xi(\nu)}{\nu}\right)\right)\overline{f}_{n-1}(z_k,\nu_k) + f_0(z,\nu).$$

Trajectory of the Monte Carlo method for $N = 2$



Complexity of the computing implementation:

1. An instability of the computing process. For more stability the generation of the large number of trajectories is needed. For computing of the solution in single point of the layer 10000 trajectories are used.

2. A slow convergence of the iterative procedure for case of the high temperature process (the case of small values N_c).

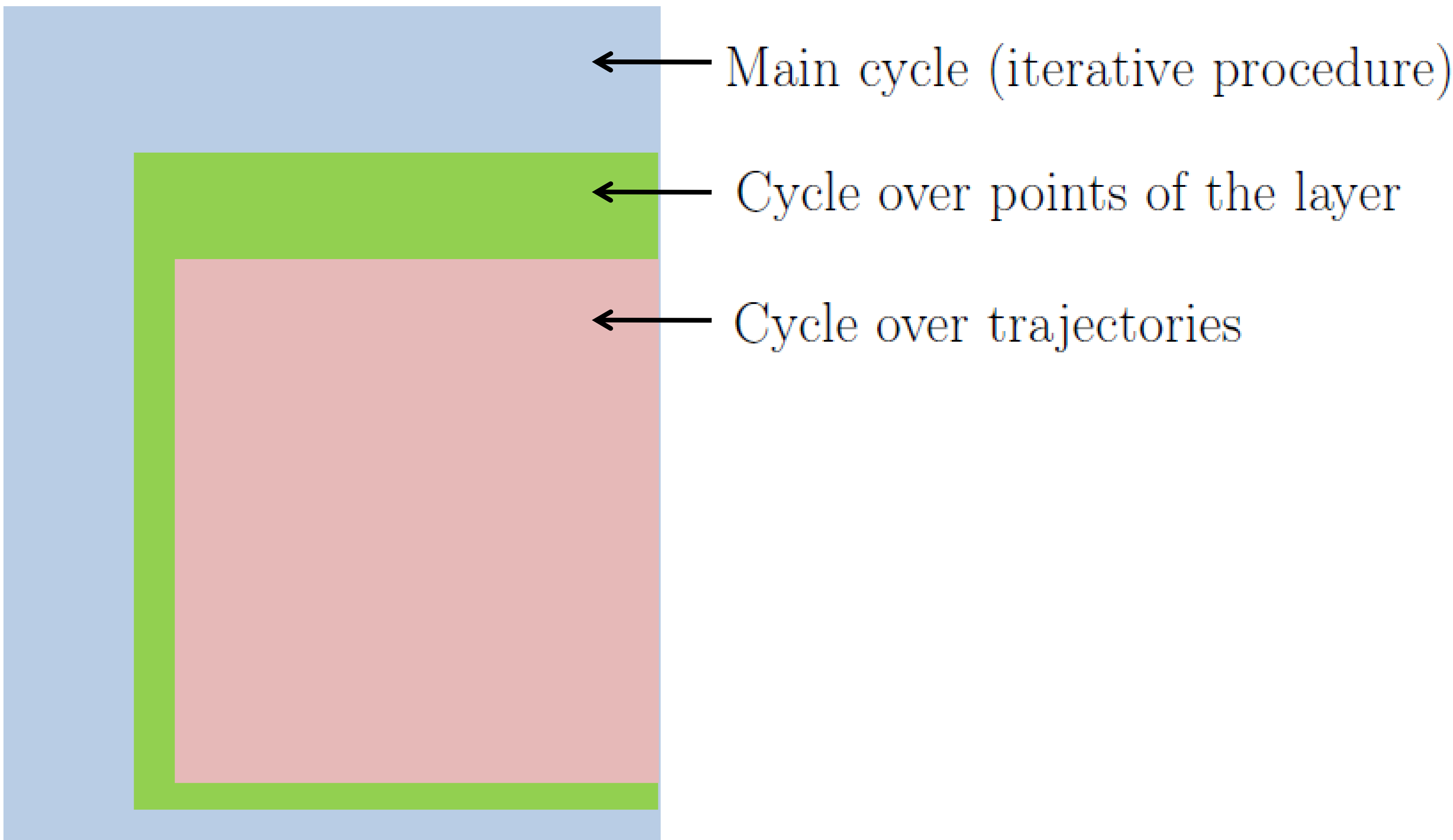
3. A number of the points in each trajectory increase as a sum of geometric series. For $N = 14$ the number of the points in single trajectory equals 7174452.

The ways of the parallelization of the computing process:

1. The calculation of the temperature at each point of the layer is performed by a separate thread.

2. The generation of each recursive trajectory of the Monte Carlo method is performed by a separate thread.

Structure of the program:



Diffusion approximation:

$$-\phi_0''(z) + 3(1-c)\phi_0(z) = 3(1-c)u^4(z), \tag{11}$$

$$\varepsilon_1\phi_0(0) - \frac{1}{2}\left(1 + \rho_1^s + \frac{4}{3}\rho_1^d\right)\phi_0'(0) = \varepsilon_1 U_1^4, \tag{12}$$

$$\varepsilon_2\phi_0(d) + \frac{1}{2}\left(1 + \rho_2^s + \frac{4}{3}\rho_2^d\right)\phi_0'(d) = \varepsilon_2 U_2^4. \tag{13}$$

$$u(z) = -\widetilde{\sigma}\phi_0(z) + C_1 z + C_2, \quad \widetilde{\sigma} = \frac{1}{3N_c}. \tag{14}$$

Numerical experiments

$$c = 0.9,\ d = 3$$

$$U_1 = 1,\ \rho_1^s = 0.1,\ \rho_1^d = 0.2,\ \varepsilon_1 = 0.7$$

$$U_2 = 0.5,\ \rho_2^s = 0.3\ ,\ \rho_2^d = 0.1,\ \varepsilon_2 = 0.6$$

$$N_c = 0.05; 0.00001$$

1. C.E. Siewert, J.R. Thomas, A computational method for solving a class of coupled conductive-radiative heat-transfer problems, J. Quant. Spectrosc. Radiat. Transfer, 45 (5) (1991) 273–281.
2. C.E. Siewert, An improved iterative method for solving a class of coupled conductive-radiative heat-transfer problems, J. Quant. Spectrosc. Radiat. Transfer, 54 (4) (1995) 599–605.

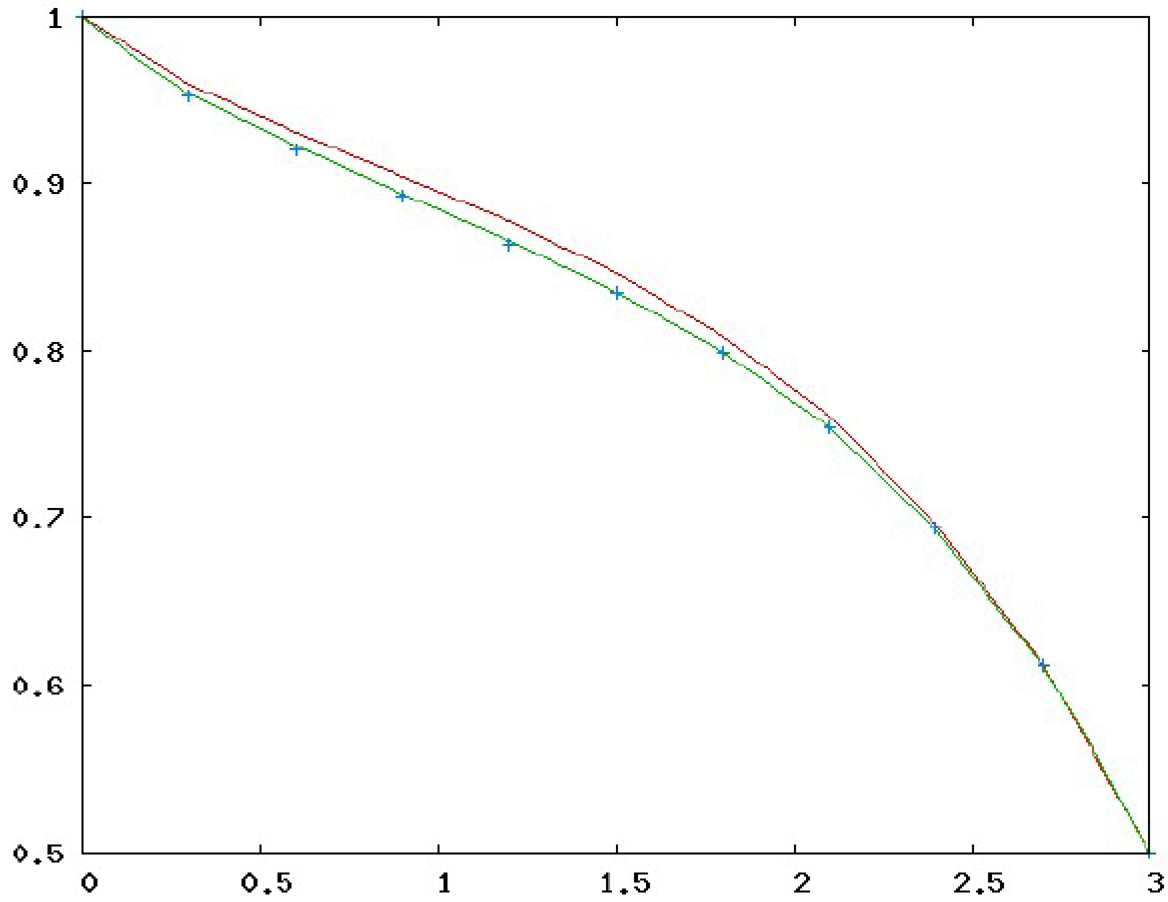


Fig. 1. The results of numerical simulation for $N_c = 0.05$ by the 20 steps of iterative algorithm based on: the Monte Carlo method, diffusion approximation in comparison with Siewert data (+)

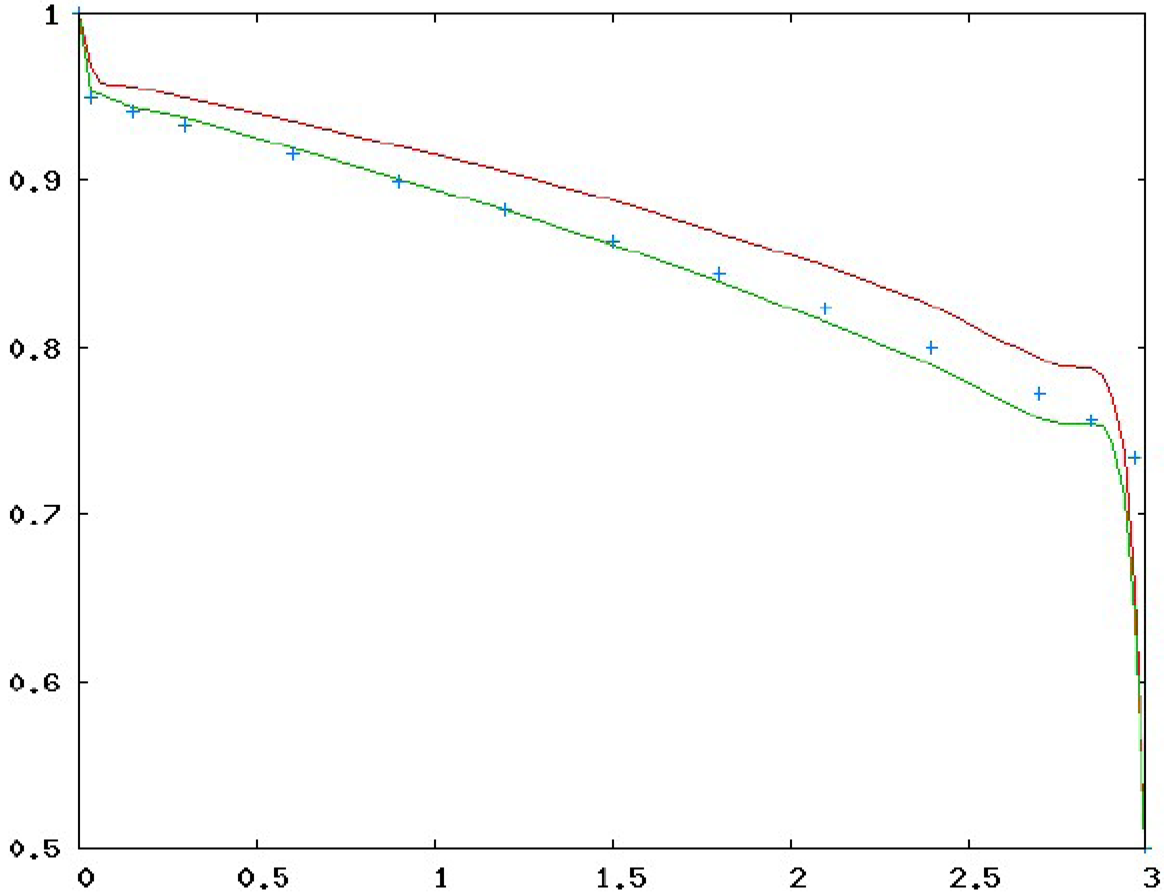


Fig. 2. The results of numerical simulation for $N_c = 0.00001$ by the 500 steps of iterative algorithm based on: the Monte Carlo method, diffusion approximation in comparison with Siewert data (+)

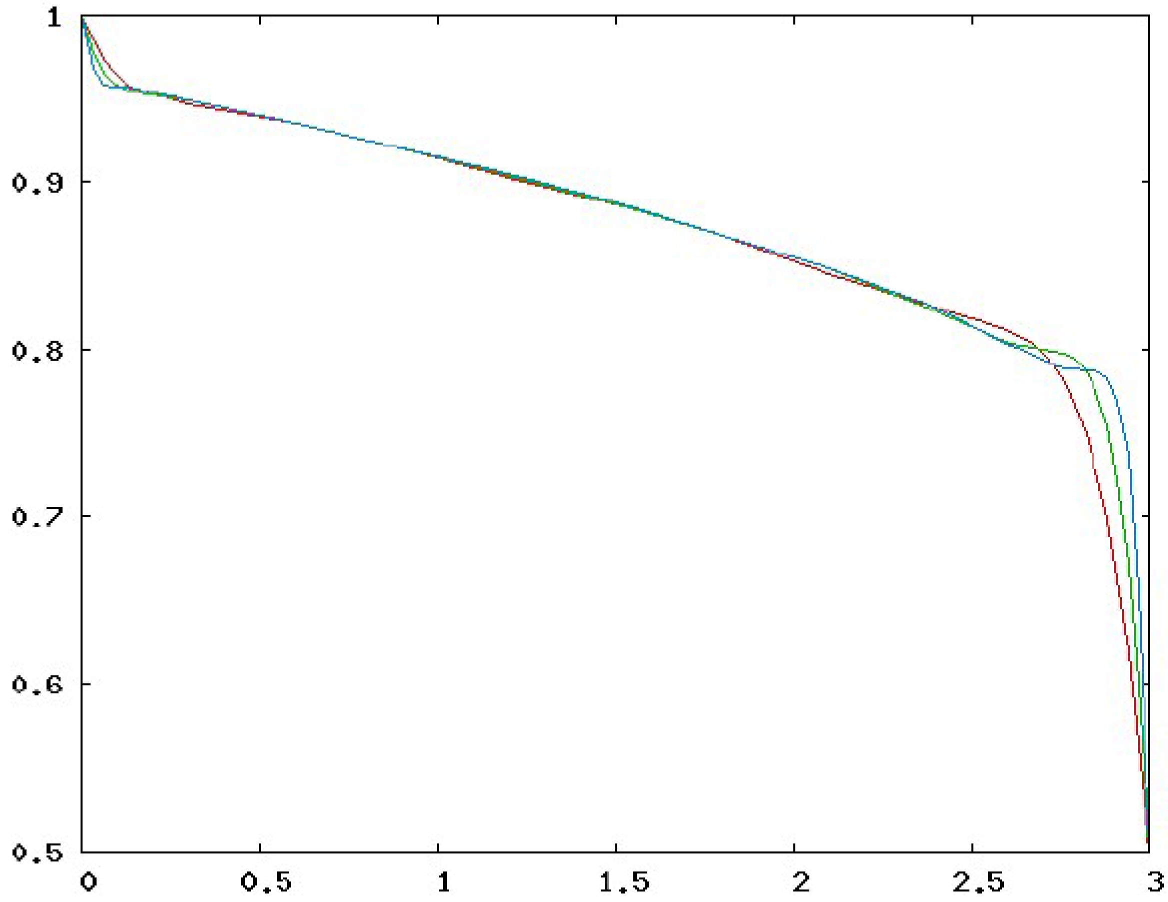


Fig. 3. The numerical experiments for $N_c = 0.00001$ which demonstrated a convergence of iterative procedure based on Monte Carlo method. The plots correspond to 50 steps , 150 steps and 500 steps of the iterative procedure.

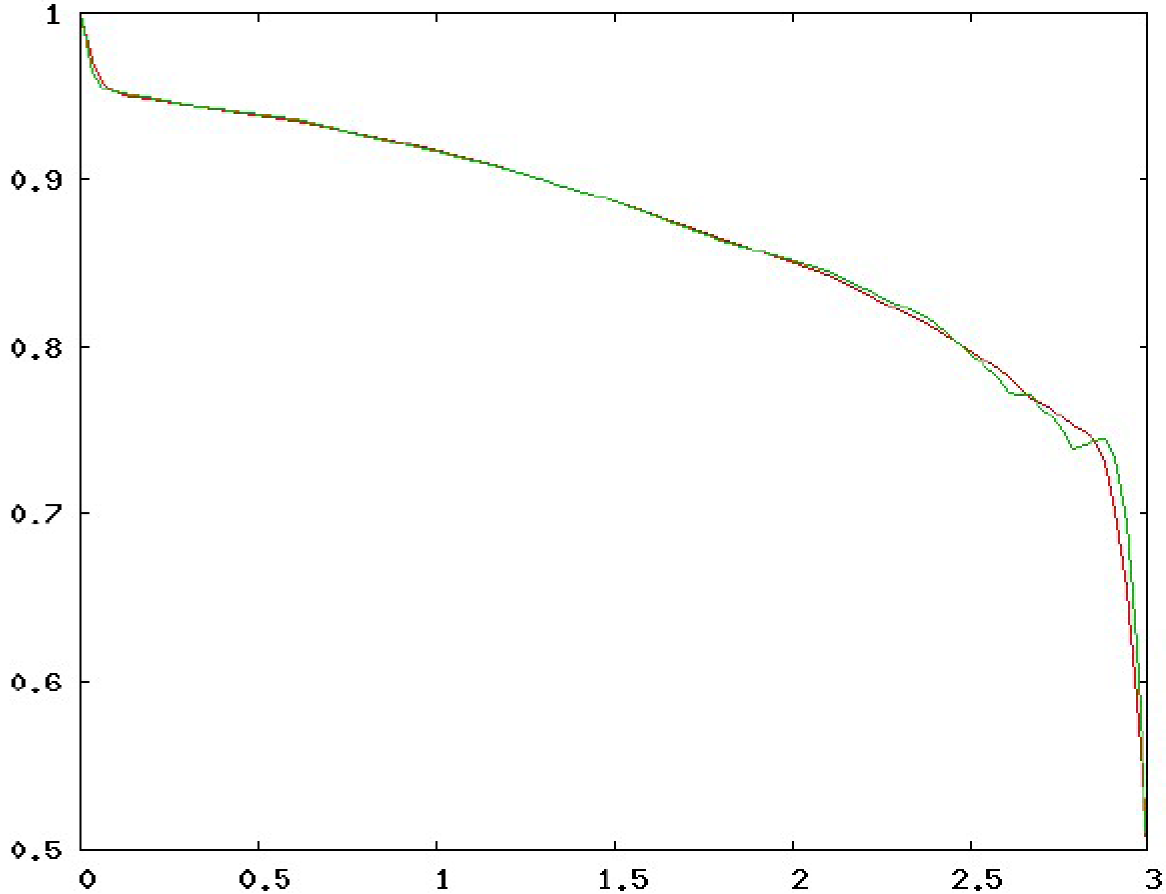


Fig. 4. The numerical experiments for $N_c = 0.00001$ which demonstrated instability of iterative procedure based on Monte Carlo method. This unsteadiness takes place for case insufficient number of trajectories. The plots correspond to 300 steps and 900 steps of the iterative procedure.

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Authors

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Introduction

Active work in determining radiological characteristics of different materials was started in the forties of the 20'th century, along with research dedicated to nuclear weapons and atomic energy. As of now large amount of experimental and theoretical data was obtained and is available in references and databases.

This database differs in that it's information is not about individual materials, but rather pairs of contacting materials. This organization scheme arose from the problem of determining a structure of heterogeneous media in x-ray tomography. In the industry this problem is related to non-damaging quality control of machine and machine parts, in medicine - analyzing damaged tissue, bones and organs.

D.S. Anikonov introduced a method for solving this problem, based on his notion of visibility measure [1] and integro-differential heterogency indicator [2]. Subsequent theoretical and numerical research is presented in works [3-33]. Among it are monographs [3-5] and Russian Federation patent [6] worth a special mention.

References

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http://imcs.dvgu.ru/struc/kmf/dbase/invisible/

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Tables of poorly visible media

Energy interval from 0.001 MeV to 20 MeV

Available materials

HYDROGEN (1)
HELIUM (2)
LITHIUM (3)
BERYLLIUM (4)
BORON (5)
SODIUM (11)
MAGNESIUM (12)
ALUMINUM (13)
SILICON (14)
PHOSPHORUS (15)
SULFUR (16)
CLORINE (17)
ARGONE (18)
POTASSIUM (19)
CALSIUM (20)
SCANDIUM (21)
TITANIUM (22)
VANADIUM (23)
CROMIUM (24)
MAMGANESE (25)

→

←

Selected materials

CARBON. GRAPHITE (6)
NITROGEN (7)
OXYGEN (8)
FLUORINE (9)
NEON (10)

Интернет 100%

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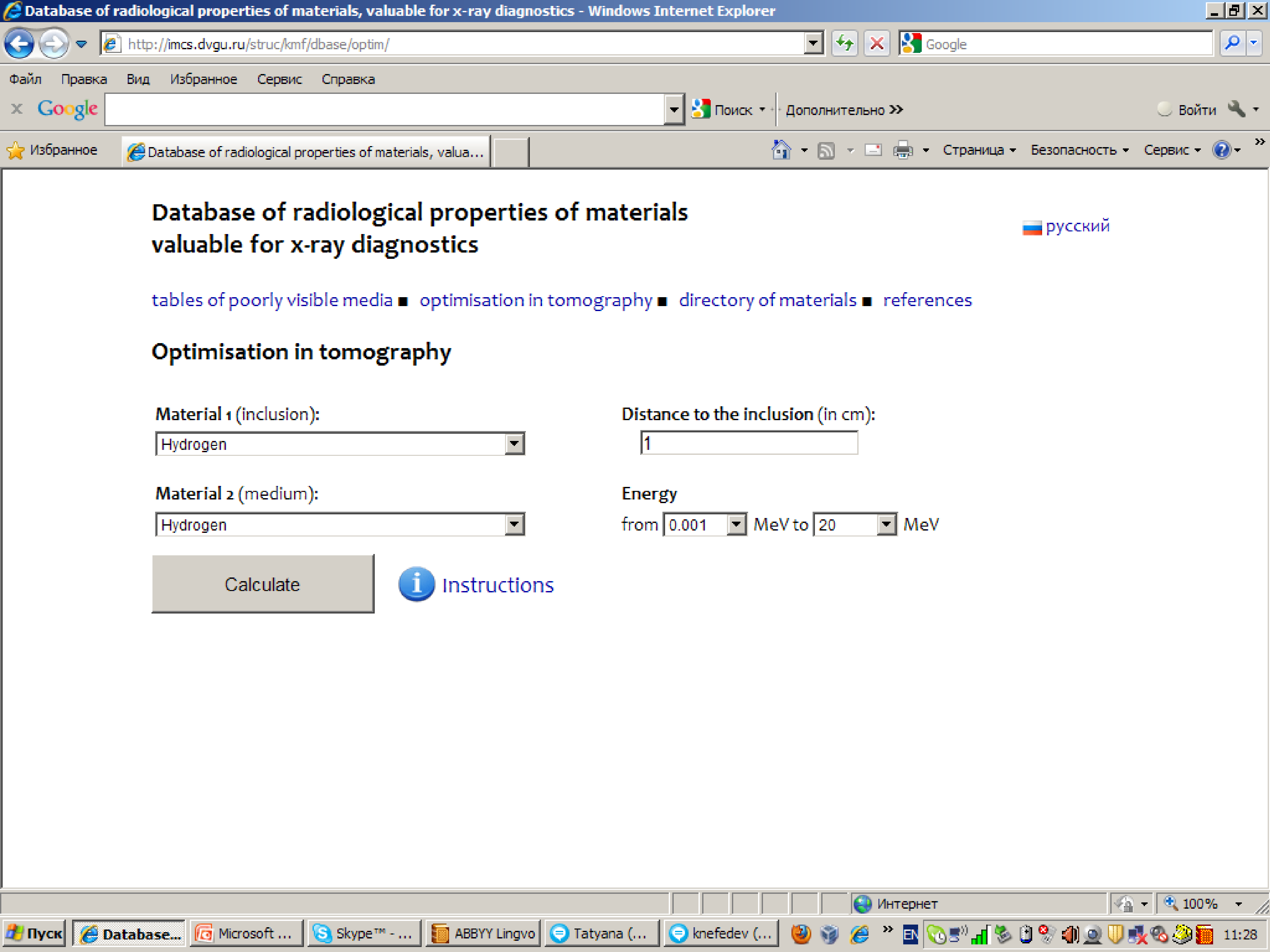
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Optimisation in tomography
Material 1 (inclusion):
Hydrogen
Distance to the inclusion (in cm):
1
Material 2 (medium):
Hydrogen
Energy
from 0.001 MeV to 20 MeV
Calculate
Instructions
Интернет
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Chemical elements

1. Hydrogen
2. Helium
3. Lithium
4. Beryllium
5. Boron
6. Carbon, graphite
7. Nitrogen
8. Oxygen
9. Fluorine
10. Neon
11. Sodium
12. Magnesium
13. Aluminum
14. Silicon
15. Phosphorus
16. Sulfur
17. Chlorine
18. Argon
19. Potassium
20. Calcium

21. Scandium
22. Titanium
23. Vanadium
24. Chromium
25. Manganese
26. Iron
27. Cobalt
28. Nickel
29. Copper
30. Zink
31. Gallium
32. Germanium
33. Arsenic
34. Selenium
35. Bromine
36. Krypton
37. Rubidium
38. Strontium
39. Yttrium
40. Zirconium

41. Niobium
42. Molybdenum
43. Technetium
44. Ruthenium
45. Rhodium
46. Palladium
47. Silver
48. Cadmium
49. Indium
50. Tin
51. Antimony
52. Tellurium
53. Iodine
54. Xenon
55. Cesium
56. Barium
57. Lanthanum
58. Cerium
59. Praseodymium
60. Neodymium

61. Promethium
62. Samarium
63. Europium
64. Gadolinium

66. Dysprosium
67. Holmium
68. Erbium
69. Thulium

71. Lutetium
72. Hafnium
73. Tantalum
74. Tungsten

76. Osmium
77. Iridium
78. Platinum
79. Gold

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Tables of poorly visible media

Energy levels from 0.001 MeV to 20 MeV, where border between SODIUM and materials NEON MAGNESIUM ALUMINUM SILICON is poorly visible

Second medium	Energy (MeV)	Coefficients for: SODIUM				Coefficients for second medium			
		Absorbtion coefficient		Attenuation coefficient		Absorbtion coefficient		Attenuation coefficient	
12. MAGNESIUM	1.07210E-3	5.253E+2	6.137E+3	5.272E+2	6.248E+3	1.359E+3		1.363E+3	
12. MAGNESIUM	1.30500E-3	4.462E+3		4.536E+3		7.846E+2	9.239E+3	7.882E+2	9.473E+3
13. ALUMINUM	1.07210E-3	5.253E+2	6.137E+3	5.272E+2	6.248E+3	2.888E+3		2.894E+3	
13. ALUMINUM	1.55960E-3	2.869E+3		2.908E+3		9.716E+2	1.033E+4	9.773E+2	1.068E+4
14. SILICON	1.07210E-3	5.253E+2	6.137E+3	5.272E+2	6.248E+3	3.304E+3		3.311E+3	
14. SILICON	1.83890E-3	1.976E+3		2.000E+3		7.153E+2	7.127E+3	7.204E+2	7.437E+3

Coefficients jump for first medium

Coefficients jump for second medium

Coefficients jump for both media

Material 1 (inclusion):
Hydrogen

Distance to the inclusion (in cm):
1

Material 2 (medium):
Silicon

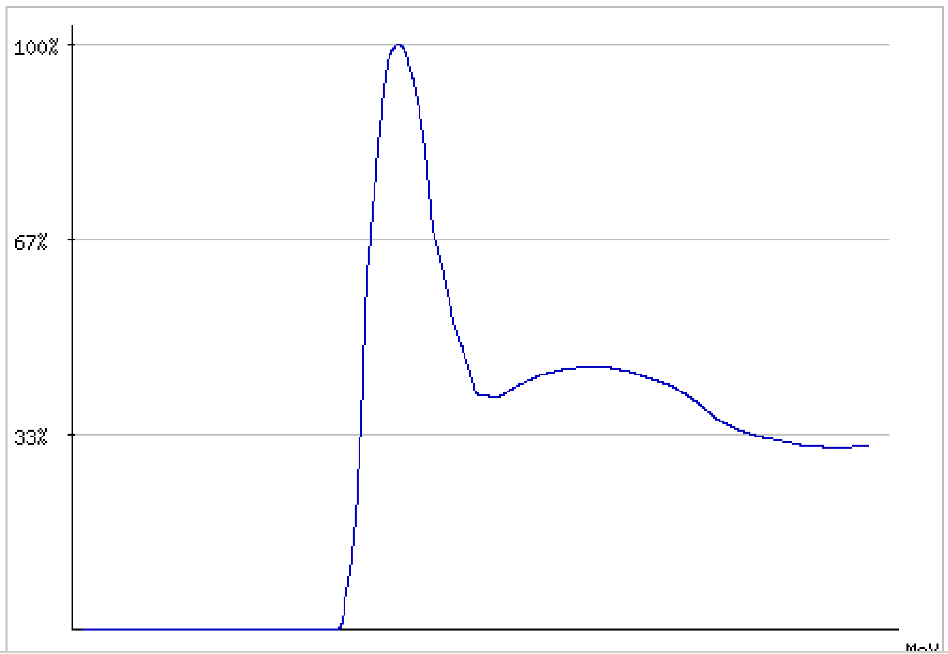
Energy
from 0.001 MeV to 20 MeV

Calculate

Instructions

Relative visibility of Hydrogen in the Silicon on the interval from 0.001 MeV to 20 MeV

Energy, MeV	Relative visibility, %
0.0010	0
0.0015	0
0.0018	0
0.0020	0
0.0030	0
0.0040	0
0.0050	0
0.0060	0
0.0080	0
0.0100	0
0.0150	0
0.0200	0
0.0300	5.48
0.0400	61.69



Thank you for attention!